FINANCIAL ECONOMETRICS

Master in Mathematical Finance

Master in Monetary and Financial Economics

Lecture 1

Jorge Caiado, PhD
CEMAPRE and Lisbon School of Economics and Management
Email: jcaiado@iseg.ulisboa.pt
Web: http://pascal.iseg.utl.pt/~jcaiado/

Lisbon 2015

<u>Lesson 1</u> – Introduction to Financial Econometrics. Some Stylized Facts of Asset Returns

Motivations

Most traditional econometric models (e.g. the CLRM) are linear in parameters. For example, a structural model can be defined as

$$Y_t = X_t'\beta + u_t, \quad \text{with}\, u_t \sim N(0,\sigma^2)$$

However, it is well known that many relationships in finance are non-linear (Examples: the trade-off between returns and risk is non-linear; the payoffs to options are non-linear).

Campbell, Lo and MacKinlay (1997) broadly define a **non-linear data generating process** as one defined by the equation

$$Y_t = f(u_t, u_{t-1}, u_{t-2}, \ldots) \ ,$$

where f is a non-linear function and u_t is an iid error. According to Campbell et al. (1997), a more specific definition of non-linear model is given by

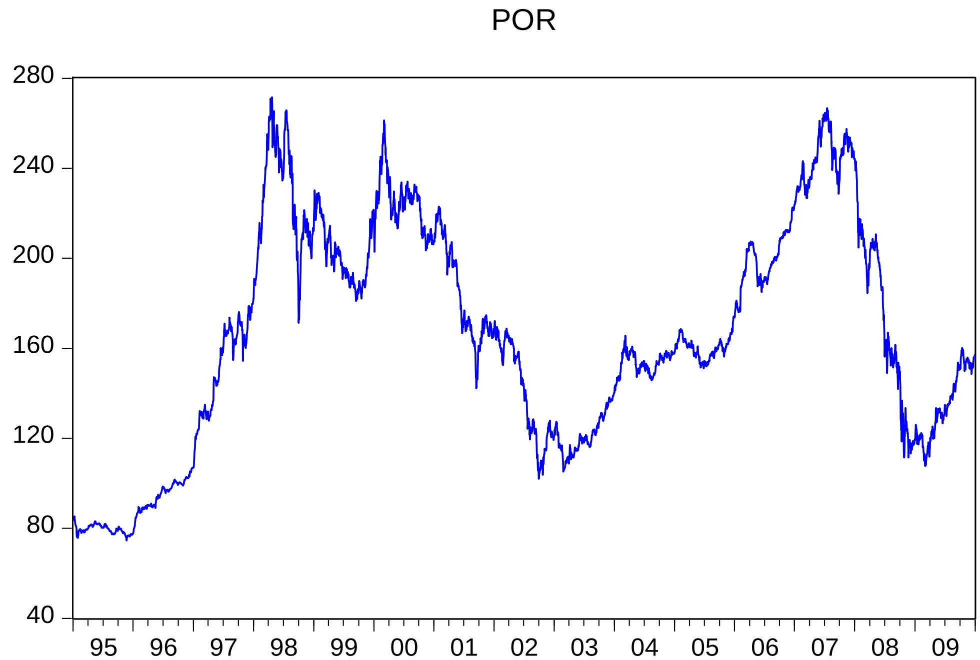
$$Y_t = g(u_{t-1}, u_{t-2}, \ldots) + u_t\sigma^2(u_{t-1}, u_{t-2}, \ldots),$$

where g is a function of past error terms only and σ^2 is a variance term. According to Campbell et al. (1997), $g(\bullet)$ is defined as being non-linear in mean and σ^2 non-linear in variance.

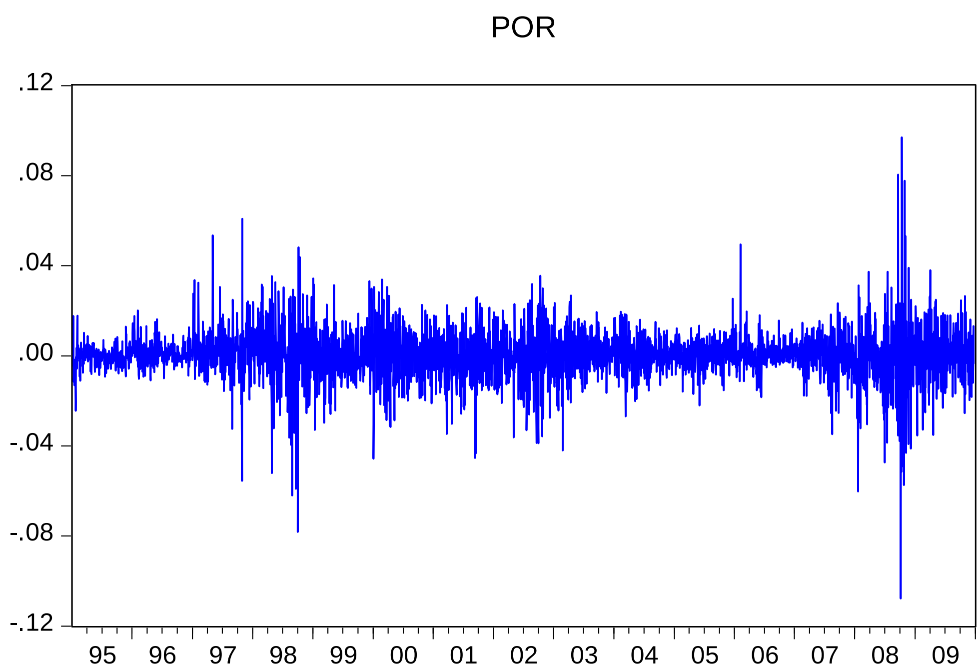
Models can be:
- Linear in mean and variance (e.g. the **ARMA models**)
- Linear in mean but non-linear in variance (e.g. **GARCH models**)
- Non-linear in both mean and variance (e.g. the hybrid threshold model with GARCH errors, see Brooks 2001)

The most popular and useful non-linear models for modeling and forecasting financial data are the **ARCH or GARCH models**, and the **switching models** (allow the behavior of a series to follow different processes at different points in time).

Prices and returns



Daily prices for the PSI20 Index over the period Jan, 2 1995 - Dec, 31 2009 (3914 obs.)



Daily returns for the PSI20 Index over the period Jan, 2 1995 - Dec, 31 2009

Simple return

Let P_t denote the price of an asset at the end of trading day *t*. The one period simple return or *"simple net return"* from date $t-1$ to date *t* is defined as

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

Holding the asset for *k* periods, the *k*-period simple return between dates $t-k$ and *t* is defined as

$$R_t(k) = \frac{P_t}{P_{t-k}} - 1$$

It is straightforward to show that $R_t(k) = \prod_{j=0}^{k-1}(1 + R_{t-j}) - 1$.

Continuously compounded return

When the capitalization of invested capital is continuous, the continuously compounded return *r* between periods 0 and 1 is obtained from equation $P_1 = P_0 e^r$

Taken natural logarithms and solving the equation $P_1 = P_0 e^r$ with respect to the variable *r*, we obtain

$$r = \ln P_1 - \ln P_0$$

The ***continuously compounded return*** or ***log return*** is then defined as

$$r_t = \ln P_t - \ln P_{t-1}$$

It can easily be shown that

$$r_t = \ln(R_t + 1)$$

For multiperiod returns, we have

$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$$

Annualized Returns

The annualized (average) return is obtained from the relation

$$P_0(1+R_t^A)^k = P_n$$

That is

$$R_t^A = \left(\frac{P_n}{P_0}\right)^{\frac{1}{k}} - 1$$

<u>Example 1</u>:

Suppose we have na investment which begins with 1.20 € and after 700 days has become 1.80 €. What is the annualized return (assume 250 trading days in a year)?

Answer*:* **15.58%**

For ***continuously compounded returns*** or ***log returns***, the annualized log return is given by:

$$r_t^A = \frac{1}{k}\ln\left(\frac{P_n}{P_0}\right)$$

In many financial applications, we are interested in calculating not only the annualized return but also the annualized volatility. Let $\bar{r} = (1/n)\sum_{i=1}^{n} r_i$ be the average of returns over the data period, the annualized return can be computed by

$$r_t^A = N\bar{r} \text{ ou } N\bar{r} \times 100\%$$

The expected annualized return is given by

$$E(r_t^A) = NE(r_t)$$

The variance of r_t^A is given by

$$Var(r_t^A) = N\sigma^2$$

Annualized volatility (measured in terms of standard deviation) is given by

$$\sqrt{N}\sigma \quad \text{or} \quad \sqrt{N}\sigma \times 100\%$$

For example, in the case of daily data, we have

$\sqrt{250}\sigma_d \times 100\%$.

In the case of weekly data, we have $\sqrt{52}\sigma_s \times 100\%$.

Portfolio Return

Let *p* a *portfolio* (consisting of *m* assets) that places weight w_i on asset *i*. Then the **simple return of *portfolio*** *p* at time t is given by

$$R_{p,t} = \sum_{i=1}^{m} w_i R_{it} ,$$

where R_{it} is the **simple return** of asset *i*.

The ***continuously compounded return of a portfolio***, however, is not a weighted average of the log returns of the assets within the portfolio. If the simple returns R_{it} are all small in magnitude, then we have $r_{pt} \approx R_{pt}$.

Relationship between simple returns and continuously compounded returns

$$r_t = \ln(1+R_t) \quad \text{and} \quad R_t = e^{r_t} - 1.$$

or

$$r_t = 100 \times \ln\left(1+\frac{R_t}{100}\right) \quad \text{and} \quad R_t = 100 \times \left(e^{\frac{r_t}{100}} - 1\right).$$

Multiperiod continuously compounded returns are additive,

$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1},$$

However, multiperiod simple returns are not,

$$R_t(k) = (1+R_t)(1+R_{t-1})\cdots(1+R_{t-k+1}) - 1$$

<u>Example 2</u>:
If the monthly log return of an asset is 5.23%, what is the corresponding monthly simple return? Answer*:* 5.37%

<u>Example 3</u>:
If the monthly prices of an asset at times 0, 1, 2 and 3 are 102.3, 107.5, 102.1 and 112.5. Compute the corresponding monthly simple and log returns. Compute the corresponding quarterly simple and log returns.